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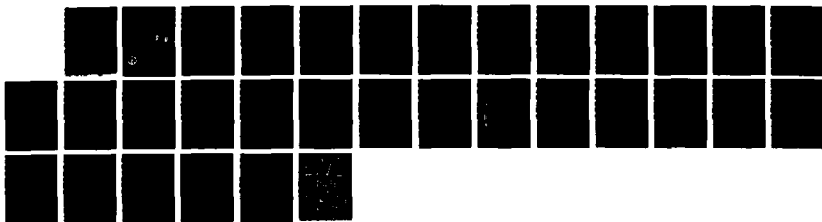
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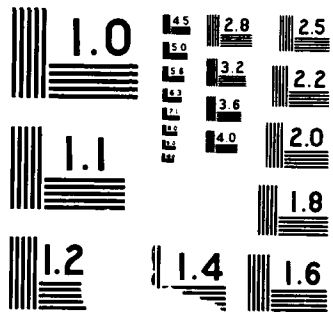
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NONLINEAR ANALYSIS OF NON-NEUTRAL PLASMAS

BY V. M. AYRES

RESEARCH AND TECHNOLOGY DEPARTMENT

30 SEPTEMBER 1987

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| <p>The purpose of this investigation is to incorporate nonlinear effects into the analysis of non-neutral plasma beams in order to accurately estimate their performance in realistic situations. Intense non-neutral plasma beams have important applications for DEW, SDI, high power microwave/millimeter wave production and particle beam guidance systems.</p> <p>Distance from Warfare Scatter Phase</p> | | | | | |
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FOREWORD

This work was performed for and funded by the Independent Research Program at Naval Surface Warfare Center under IR task number 7R01AB313, Non-Linear Analysis of Non-Neutral Plasmas. This is an investigative study undertaken to develop a nonlinear analysis of electron and/or ion beams for accurate prediction of their behavior as a function of time. The configuration chosen for the first application of this analysis was the TE_{11} interaction with an axis-rotating electron beam, in a cylindrical waveguide, operated in the amplifier mode. We have successfully derived the efficiency of this interaction in terms of experimentally controllable parameters, and have optimized it with respect to wave frequency.

Approved by:

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CHAPTER 1

INTRODUCTION

GENERAL BACKGROUND

The project, Nonlinear Analysis of Non-Neutral Plasmas, is ultimately concerned with the uses of electron and/or ion beams in the generation of high power microwave and millimeter waves or, conversely, the production of high brightness particle beams. Military applications of high power microwaves include microwave jamming, radar tracking, and low altitude satellite neutralization. High brightness particle beams are under investigation as potential weapons for application as part of the Strategic Defense Initiative, and may also be used for particle beam guidance systems.

The very high power/brightness requirements of these devices necessitates the development of a new generation of highly efficient electron/ion beam sources, beyond the level of those which are currently available. In the past, most attempts to build such devices have relied on empirical data only, a method which has often been costly, and which certainly carries no guarantee of success. It is strongly felt within the particle beam community that the development of the next generation of electron/ion beam devices, with their more stringent requirements for power, gain, and frequency, will only be possible with a much more complete understanding of the interaction mechanisms than has been achieved to date.

These mechanisms, involving the interaction of the beam and the rf field, differ from problem to problem, but they have a point in common: that they are usually intrinsically nonlinear. An examination of Figure 3-3 of this report illustrates the point. In a description of the growth of the wave field amplitude in a microwave generation problem, there is a region of linear growth marked on the graph, but there is also a region of much larger nonlinear growth. Valuable information about the saturation point of the interaction, which fixes the maximum efficiency of the device, is only accessible by studying the evolution of the wave field amplitude through the nonlinear regime.

Many examples such as this one have motivated the nonlinear analysis of problems involving beam-rf field interactions. There are two approaches to nonlinear analysis. One is to perform a computer simulation of the problem. This involves combining given initial conditions with the electron/ion equations of motion and then solving these equations in a recursive fashion. There are two major obstacles. One is that the number of particles that even the best modern computers can handle falls far short of the number of particles in even a low current beam. The other is that the evolution of the system becomes a black box with all its basic physics hidden. Optimization of results is reduced to trial-and-error manipulation of the initial conditions.

The second approach to nonlinear analysis is to proceed analytically as far as possible, and then to introduce approximations into the solution of the equations of motion in such a way that the important features of the beam-rf field interaction are unaffected. This is the approach which we have followed. Our goal has been to develop a nonlinear analysis of electron and/or ion beams, for accurate prediction of their behavior; most importantly, their behavior as a function of time. Our approach involves the approximate solution of the electromagnetic field equations, retaining nonlinear features as they pertain to the time development of electromagnetic field quantities and beam parameters. This approach has proved very successful in the analysis of the first configuration chosen for examination, the cylindrical waveguide TE_{11} interaction with an axis-rotating electron beam, operating in the amplifier mode.

CONFIGURATION: TE_{11} INTERACTION WITH AXIS-ROTATING BEAM

The TE_{11} mode excitation is of great practical interest in microwave generation since it is the dominant mode in a cylindrical circuit. The optimum beam configuration for exciting the TE_{11} mode is known to be an axis-rotating beam^{1,2} which can be produced by injection of a hollow beam through a cusp magnetic field. In preliminary experiments powerful radiation with multikilowatt output at the fundamental electron cyclotron frequency was observed in the NSWC Cusptron device. This prompted a detailed theoretical investigation of the efficiency of this configuration.

APPROACH

To theoretically study the efficiency of TE_{11} mode generation by an axis-rotating beam via the negative mass instability, a nonlinear analysis is developed. Other configurations analyzed by nonlinear analysis include the gyrotron in a cylindrical waveguide³ and in a parallel plate waveguide⁴; and an axis-rotating beam in a multivane structure⁵. We assume that phase trapping of the electrons in the wave fields is the dominant saturation mechanism. The investigation of the problem starts with solving the nonlinear coupled equations of single electron motion in the radiation fields. Next, the response of the radiation fields to the current induced by the electron bunching is analyzed using conservation of energy. Then the results are averaged over the initial random phase distribution of the electron's transverse velocity to obtain the collective effect on the wave fields by the electrons. The result of the nonlinear analysis performed in this study is the derivation of a single nonlinear third-order differential equation for the wave field amplitude as a function of time. From knowledge of the temporal evolution of the wave field, the efficiency of the interaction may be directly calculated.

REPORT OVERVIEW

In Chapter 2 we develop the nonlinear analysis necessary to study the wave field amplitude as a function of time. We derive a third-order differential equation which may be solved numerically to obtain the temporal evolution of the wave field over its entire range, through the linear, nonlinear, and saturation regimes. In Chapter 3 the numerical solution of the full equation is carried out and the efficiency of the interaction is studied parametrically as a function of the beam energy and beam current, the external magnetic field, and the waveguide radius. The efficiency is also optimized as a function of wave frequency. Our conclusions are presented in Chapter 4.

CHAPTER 2

NONLINEAR ANALYSIS OF TE_{11}
INTERACTION WITH AXIS-ROTATING ELECTRON BEAM

FIELDS AND GEOMETRY

The geometry of the problem is as shown in Figure 2-1. The equilibrium configuration consists of a non-neutral electron layer that is infinite in extent and aligned parallel to an applied magnetic field $B_0 \hat{z}$. The electron layer is accomplished by passing a hollow beam through an ideal cusp magnetic field. The investigation of the problem starts with solving the nonlinear coupled equations of single electron motion in the electromagnetic fields. The motion of an electron in the external magnetic field and the waveguide mode fields is governed by the set of coupled nonlinear equations

$$\frac{d\vec{r}}{dt} = \frac{\vec{P}}{\gamma m_0} , \quad (2-1)$$

$$\frac{d\vec{P}}{dt} = -e[\vec{E} + \frac{1}{c} \vec{v} \times (\vec{B} + B_0 \hat{z})] , \quad (2-2)$$

$$\frac{d\gamma}{dt} = -\frac{e}{m_0 c^2} \vec{E} \cdot \vec{v} , \quad (2-3)$$

where $\gamma = (1 + \vec{P}^2 / (m_0^2 c^2))^{1/2}$, and $\vec{P} = \gamma m_0 \vec{v}$.

The wave fields $\vec{E}(\vec{x}, t)$ and $\vec{B}(\vec{x}, t)$ of the TE_{mn} mode inside a circular cylinder of radius R_w are given by,

$$E_r = -E_0(t) \frac{m J_m(\gamma_{mn} r)}{\gamma_{mn} r} \cos(m\phi + k_z z - \omega t) ,$$

$$E_\phi = E_0(t) J'_m(\gamma_{mn} r) \sin(m\phi + k_z z - \omega t) ,$$

$$E_z = 0 ,$$

$$B_r = -\frac{k_z c}{\omega} E_\phi ,$$

$$B_{\phi} = \frac{k_z c}{\omega} E_r ,$$

$$B_z = \frac{\gamma_{mn} c}{\omega} E_o(t) J_m(\gamma_{mn} r) \cos(m\phi + k_z z - \omega t) , \quad (2-4)$$

where γ_{mn} is the n^{th} root of the equation: $J'_m(\gamma_{mn} R_w) = 0$. We assume that the electromagnetic fields are those of a vacuum waveguide, but with amplitudes that vary with time. The implications of this assumption are discussed in Chapter 4.

Rather than approach this problem as a computer simulation we will try to identify the source of the greatest contribution to the wave field growth. The exchange of energy between the wave field and the beam will be greatest when the wave frequency, ω , is near the Doppler shifted electron cyclotron frequency, $m\Omega + k_z v_z$, where $\Omega = \Omega_o/\gamma$, and $\Omega_o = eB_o/mc$. In this paper we examine the fundamental, $m = 1$, case. We note that the quantity

$$\Delta\omega(t) = \omega - \{k_z v_z(t) - \Omega(t)\} \quad (2-5)$$

essentially defines the degree of resonance present. In this approach, an average is taken over the initial random phase in such a way that quantities related to the fast cyclotron motion are averaged out.

A self-consistent trajectory is employed to describe the location of a single electron within the rotating electron layer,

$$r(t) = R_L(t) , \quad (2-6)$$

$$\phi(t) = \phi_o + \psi(t) + \int_0^t \Omega(t') dt' . \quad (2-7)$$

$R_L(t)$ is the time-dependent Larmor radius of the rotating electron. The time-dependent phase $\phi(t)$ is composed of three parts: the initial phase ϕ_o , the usual term due to the electron cyclotron motion in the external B_z field, and a slowly varying phase $\psi(t)$ due to the interaction of the wave field with the electron. By substituting Equations (2-1), (2-4), (2-5), and (2-6) into Equations (2-2) and (2-3), a total of four equations are derived for the four slowly varying quantities: the total relativistic factor $\gamma(t)$ (energy = γmc^2), the axial velocity $v_z(t)$, the slowly varying phase $\psi(t)$, and the generalized Larmor radius $\alpha(t) = \gamma_{11} R_L(t)$. Explicitly, these equations are

$$\frac{d\gamma(t)}{dt} = -E_o(t) c_2 \left(\frac{\Omega_o}{\gamma_{11} c} \right)^2 \omega f(\alpha) \sin \Phi , \quad (2-8)$$

$$\frac{dv_z(t)}{dt} = E_o(t) c_2 \left(\frac{\Omega_o}{\gamma_{11} c} \right)^2 \omega \frac{B_o}{k_z \gamma_{11}} \left(k_z v_z - \frac{k_z^2 c^2}{\omega} \right) f(\alpha) \sin \Phi , \quad (2-9)$$

$$\frac{d\alpha(t)}{dt} = -E_o(t) c_1 J'_1(\alpha) \sin \Phi, \quad (2-10)$$

$$\frac{d\Phi(t)}{dt} = -\Delta\omega(t) \quad (2-11)$$

$$- E_o(t) \left\{ \frac{\gamma_{11} c}{B_o} \frac{\Delta\omega(t)}{\omega} J_1(\alpha) + c_1 \left[\frac{1}{\alpha} \frac{d(\alpha J'_1(\alpha))}{d\alpha} \right] \right\} \cos \Phi,$$

where $f(\alpha) = \alpha J'_1(\alpha)/\gamma$, $c_2 = \gamma_{11} c/(B_o \omega)$, and $c_1 = c_2(\omega - k_z v_z)$.

The quantity $\Phi(t)$ is the relative phase $\Phi(t) = \phi(t) + k_z z(t) - \omega t$, which may be expressed as $\Phi(t) = (\phi_o + \psi(t)) - \int_0^t \Delta\omega(t') dt'$ where $\Delta\omega(t)$ is given by Equation (2-5).

GRAZING CONDITION

The efficiency is expected to be highest at or near grazing, therefore we limit our attention to the case where the grazing condition, $v_z = k_z c^2/\omega$, is valid. Upon application of the grazing condition, Equation (2-8) for $dv_z(t)/dt$ equals zero; thus the grazing condition implies that $v_z(t) = v_{zo}$ at all times. From Equation (2-11), $\Delta\omega(t)$ may be explicitly evaluated. This is easily shown to be

$$\Delta\omega(t) = \Delta\omega_o + \left(\omega_o - \frac{k_z^2 c^2}{\omega} \right) \left(\frac{\gamma - \gamma_o}{\gamma} \right), \quad (2-12)$$

where $\Delta\omega_o = \omega - \omega_o$, and $\omega_o = \Omega_o/\gamma_o + k_z v_{zo}$, although the grazing condition must be used to obtain the result in this form. Integrating Equation (2-8) to obtain $\gamma - \gamma_o$, Equation (2-12) becomes

$$\Delta\omega(t) = \Delta\omega_o - \left\{ \left(\omega_o - \frac{k_z^2 c^2}{\omega} \right) \int_0^t dt' E_o(t') \omega c_2 \left(\frac{\Omega_o}{\gamma_{11} c} \right)^2 f(t') \sin \Phi(t') \right\}. \quad (2-13)$$

CONSERVATION OF ENERGY

At this point, Equations (2-7)-(2-10) define the effect of the wave field on a single electron. To find the reciprocal effect of the ensemble of beam electrons on the wave field an additional field equation is required. Conservation of energy (Poynting equation), with the fields in Equation (2-4) is adequate for this purpose as Equations (2-7)-(2-10) involve only the amplitude of the wave field. It is straightforward to show that conservation of energy implies that

$$n_o m_o c^2 \left\langle \frac{d\gamma(t)}{dt} \right\rangle + \frac{\epsilon}{8\pi} E_o(t) \frac{dE_o(t)}{dt} = 0 . \quad (2-14)$$

where n_o is the electron beam density averaged over the cross-section of the waveguide, $\langle \rangle$ stands for the average over the initial random phase distribution of the electron beam, and ϵ is given by

$$\epsilon = \left(1 - \frac{1}{\gamma_{11}^2 R_W^2} \right) J_1^2(\gamma_{11} R_W) . \quad (2-15)$$

Upon substitution of Equation (2-7) for $d\gamma(t)/dt$ into Equation (2-14) one finds that

$$\langle f \sin(\Phi) \rangle = \frac{1}{P} \frac{dE_o(t)}{dt} , \quad (2-16)$$

where $P = 4\pi n_o m_o \Omega_o^2 c / (\epsilon \gamma_{11} B_o)$.

PHASE AVERAGE

We next determine the phase averaged quantity $\langle f \sin \Phi \rangle$ in terms of $E_o(t)$ and its derivatives using the two remaining equations, Equations (2-10) and (2-11). Details of the calculation are provided in Appendix A. Three approximations are made. First, it is assumed that the effect of beam energy depletion can be neglected. This is consistent with our assumption that phase trapping is the dominant saturation mechanism. Second, it is assumed that wave frequency is on the order of (but not equal to) the initial electron cyclotron frequency. The third approximation is that only linear and nonlinear terms of the lowest order are retained in the performance of the phase averaging. Details of the phase averaging procedure are provided in Appendix B and discussed in Chapter 4.

NONLINEAR DIFFERENTIAL EQUATION AND EFFICIENCY

The final result is now derived. Using Equation (A-7) for $\langle f \sin \Phi \rangle$ and Equation (A-8) for $\langle f \cos \Phi \rangle$, together with Equation (2-16), the following nonlinear third-order equation for the wave field amplitude $E_o(t)$ as a function of time, t , results,

$$\begin{aligned} \frac{d^3 E_o}{dt^3} + \left\{ \frac{R_o}{\Delta \omega_o} \right\} I_c E_o \frac{dE_o^2}{dt^2} + \left\{ \Delta \omega_o^2 - R_1 E_o^2 + R_o E_o I_s + PD_1 \right\} \frac{dE_o}{dt} = \\ \left\{ PD_o \right\} \left\{ \Delta \omega_o + \langle \Delta \omega \rangle \right\} I_c - \left\{ \frac{PR_o}{\Delta \omega_o} \right\} I_c E_o \left\{ D_1 E_o + D_o I_s \right\}, \end{aligned} \quad (2-17)$$

where

$$\begin{aligned} D_o &= \frac{c_4}{2} \frac{(\alpha_o J'_{1o})^2}{\gamma_o^3}, \\ D_1 &= \frac{c_1}{2} \left\{ \frac{J_{1o}^2}{\gamma_o} \left(\frac{c_3}{\gamma_o^2} + 1 \right) + \frac{2\alpha_o J'_{1o} J''_{1o}}{\gamma_o} \right\}, \\ R_o &= \frac{3}{4} c_1 c_3 c_4 \left(\frac{\Omega_o}{\gamma_o \gamma_{11} c} \right)^2 \left(\frac{\alpha_o J'_{1o}}{\gamma_o} \right)^2, \\ R_1 &= c_1^2 \left\{ \frac{1}{\alpha_o} \frac{d}{d\alpha_o} \left(J'_{1o} \frac{d(\alpha_o J'_{1o})}{d\alpha_o} \right) \right. \\ &\quad \left. + \frac{3}{4} \left(\frac{\Omega_o}{\gamma_o \gamma_{11} c} \right)^2 c_3 \left[3 \left(\frac{\Omega_o}{\gamma_o \gamma_{11} c} \right)^2 (\alpha_o J'_{1o})^2 - 4 J'_{1o} \frac{d(\alpha_o J'_{1o})}{d\alpha_o} \right] \right\}, \end{aligned}$$

$$I_c = \int_0^t dt' E_o(t') \cos \langle \Delta \Phi(t - t') \rangle,$$

$$I_s = \int_0^t dt' E_o(t') \sin \langle \Delta \Phi(t - t') \rangle, \quad (2-18)$$

with $\langle \Delta \Phi(t - t') \rangle = -\int_{t'}^t \langle \Delta \omega(t'') \rangle dt''$ (remaining constants are given in Appendix A).

Because the numbers involved can be large, it is convenient to normalize this by setting $X(\xi) = \left[\epsilon / (16\pi n_m c^2) \right]^{1/2} E_o(t)$, a small number, and $\xi = \Delta \omega_o t$, a slow time scale. Then Equation (2-17) takes its final form

$$\begin{aligned} \frac{d^3 X}{d\xi^3} + 2\delta_o \bar{I}_c X \frac{d^2 X}{d\xi^2} + \left\{ 1 - \beta_o X^2 + \beta_1 X \bar{I}_s + d \right\} \frac{dX}{d\xi} = \\ b \left\{ 1 - g[X^2(\xi) - X^2(0)] \right\} \bar{I}_c - \delta_o \bar{I}_c X \left\{ 2dX + b\bar{I}_s \right\}, \end{aligned} \quad (2-19)$$

where

$$\begin{aligned} \delta_o &= \frac{R_o}{2\Delta\omega_o^3} \left(\frac{16\pi n_o m_o c^2}{\epsilon} \right), & \beta_1 &= 2\delta_o, & \beta_o &= \frac{R_1}{\Delta\omega_o^2} \left(\frac{16\pi n_o m_o c^2}{\epsilon} \right), \\ d &= \frac{2PD_1}{\Delta\omega_o^2}, & b &= \frac{2PD_o}{\Delta\omega_o^3}, & g &= \frac{\omega - k_z v_{zo}}{\gamma_o \Delta\omega_o}, \end{aligned} \quad (2-20)$$

and

$$\begin{aligned} \tilde{I}_s &= -\int_0^\xi d\xi' X(\xi') \sin \int_\xi^\xi, [1 - 2g(X^2(\tilde{\xi}) - X^2(0))] d\tilde{\xi}, \\ \tilde{I}_c &= \int_0^\xi d\xi' X(\xi') \cos \int_\xi^\xi, [1 - 2g(X^2(\tilde{\xi}) - X^2(0))] d\tilde{\xi}. \end{aligned} \quad (2-21)$$

The efficiency of the interaction may now be calculated according to^{4,6}

$$\eta = \frac{2(X_s^2 - X_o^2)}{\gamma_{o1} - 1}, \quad (2-22)$$

where X_s is the value of the normalized electric field amplitude at the point of saturation. In Chapter 3, Equations (2-19) and (2-22) will be solved numerically to determine the optimized efficiency as a function of various parameters: beam energy and beam current, external magnetic field, and waveguide radius.

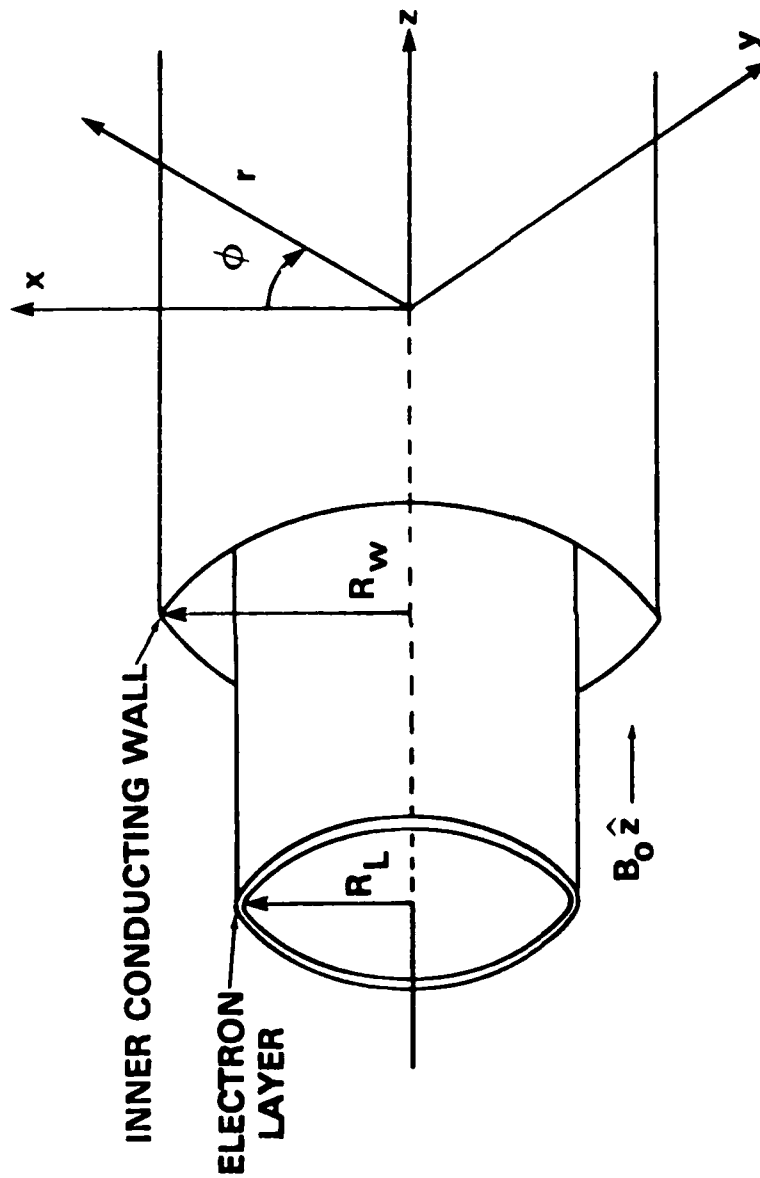


FIGURE 2-1. EQUILIBRIUM CONFIGURATION AND COORDINATE SYSTEM.

CHAPTER 3

NUMERICAL RESULTS

The TE_{11} interaction of fast axis-rotating electron beams in a cylindrical waveguide may now be analyzed using Equation (2-19) to determine the evolution of the wave field through the linear and nonlinear regimes up to the point of saturation, and Equation (2-22) to evaluate the corresponding efficiency.

We choose the initial conditions from among the experimentally relevant parameters. These are the beam energy E_k , the beam current I , the external magnetic field B_o , and the waveguide radius R_W .

It is important to solve for the efficiencies within the limits of the assumptions used to produce Equation (2-19). We examine the consequences of our assumption that the grazing condition is always met before proceeding to the efficiency calculation.

CONSEQUENCES OF GRAZING CONDITION

The grazing condition is satisfied at the point where the beam dispersion curve ω_b tangentially grazes the wave dispersion curve ω_w ,

$$\begin{aligned}\omega_b &= \frac{\Omega_o}{\gamma_o} + k_z v_{zo}, \\ \omega_w &= \left(\omega_{co}^2 + c^2 k_z^2 \right)^{1/2},\end{aligned}\tag{3-1}$$

with

$$\omega_{co} = 5.523 \times 10^{10} / R_W,\tag{3-2}$$

for the TE_{11} interaction. Grazing occurs when

$$0 = v_{zo}^2 - \frac{c^2}{\omega_{co}^2} \left(\omega_{co}^2 - \frac{\Omega_o^2}{\gamma_o^2} \right).\tag{3-3}$$

Writing this in terms of B_o , E_k , and R_W leaves the grazing condition expressed in terms of the chosen parameters,

$$\alpha^2 = \frac{\left(\frac{R_W \Omega_0}{\omega_{co}}\right)^2 - 1}{\left(\frac{E_k}{m_0 c^2} + 1\right)^2 - \left(\frac{R_W \Omega_0}{\omega_{co}}\right)^2}, \quad (3-4)$$

where $\alpha = v/v_z$. This equation is only true for certain combinations of the parameters E_k , B_0 , and R_W . In any solution of Equation (2-19) for the wave field amplitude $X(\xi)$ (the normalized $E_0(t)$), α is first determined for a given R_W and B_0 so that the grazing condition, Equation (3-4), is satisfied.

Figure 3-1 shows the behavior of α versus R_L/R_W , along with the corresponding values for β and β_z . This behavior has implications for the efficiency calculation. Although α can asymptotically approach infinity, it is clear that there will be no increase in efficiency beyond the point where β_z goes to zero and β reaches its maximum, i.e., the point where all the initial z energy is assumed to go into rotational energy. Therefore, while R_W and B_0 may both be independently varied, application of the grazing condition will link them together and limit the effect of their individual variation on the efficiency to a maximum at the point where $\beta_z \approx 0$. This point is illustrated in Figure 3-2, a plot of efficiency versus α for a 30.0 keV, 3.5 Amp beam in a 460.0 Gauss external magnetic field.

OPTIMIZED EFFICIENCIES

We may now numerically solve Equation (2-19) in a self-consistent manner. A 30.0 keV, 3.5 Amp beam in a 460.0 Gauss external magnetic field is chosen for study. Additional studies, to be reported elsewhere, indicate that changes in E_k and I do not greatly change the results to be presented here. Variations in B_0 and R_W have been discussed in the previous sub-section; for a 460.0 Gauss field, an $R_W = 7.227$ cm results in the best possible efficiency.

A Runge-Kutta scheme was employed to evaluate $X(\xi)$ versus ξ , and for most combinations of parameters a step size of $\Delta\xi = 0.0001$ was adequate. A single efficiency calculation involving 2000 steps takes approximately 3.953 CP seconds on a CDC CYBER 720 mainframe computer.

In Figure 3-3 we display a typical result obtained by integrating Equation (2-19). It shows the evolution of the wave field amplitude for a 30.0 keV, 3.5 Amp beam in an external magnetic field of 460.0 Gauss with $R_W = 7.227$ cm. The linear, nonlinear, and saturation regions are indicated on the graph. The point of saturation is clearly defined at $X_s = 0.0517$. The corresponding efficiency is 9.11%.

It is desired to perform not only a single efficiency calculation, but an efficiency optimization study. We choose to optimize the efficiency with respect to wave frequency as that is an experimentally variable parameter. To determine an appropriate parameter range for ω we utilize earlier results from the linear theory for this configuration^{7,8}. The result of this analysis is an optimized

efficiency as a function of ω .

In Figure 3-4, a plot of efficiency η as a function of frequency \underline{f} ($\underline{f} = \omega/2\pi$) is shown. The efficiency is clearly optimized at $\underline{f} \approx 1.233$ GHz, with an efficiency of 9.11%.

The curve itself is likely to be artificially broadened by the necessary use of an approximation, $k_z \approx \omega v_{z0}/c^2$, in regions where $\Delta\omega_0$ is not small. Studies are currently underway in which the grazing condition assumption itself is removed.

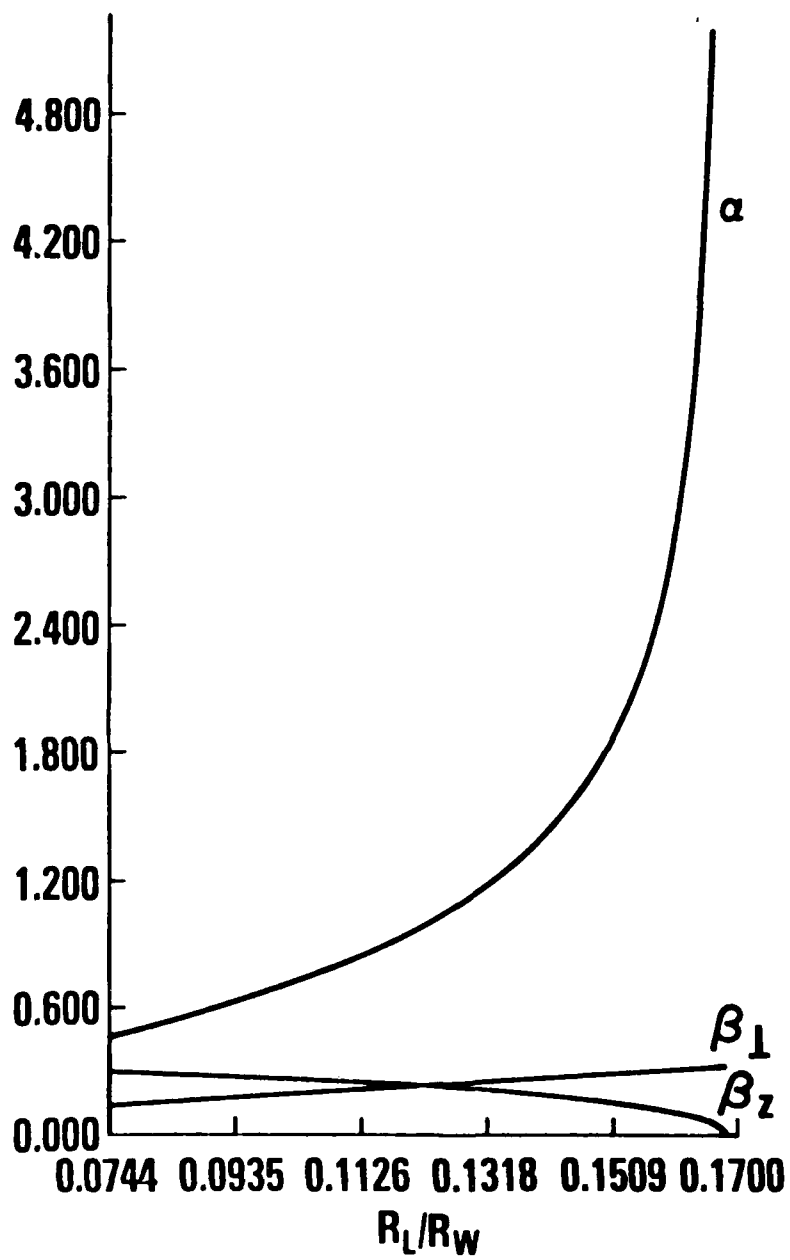
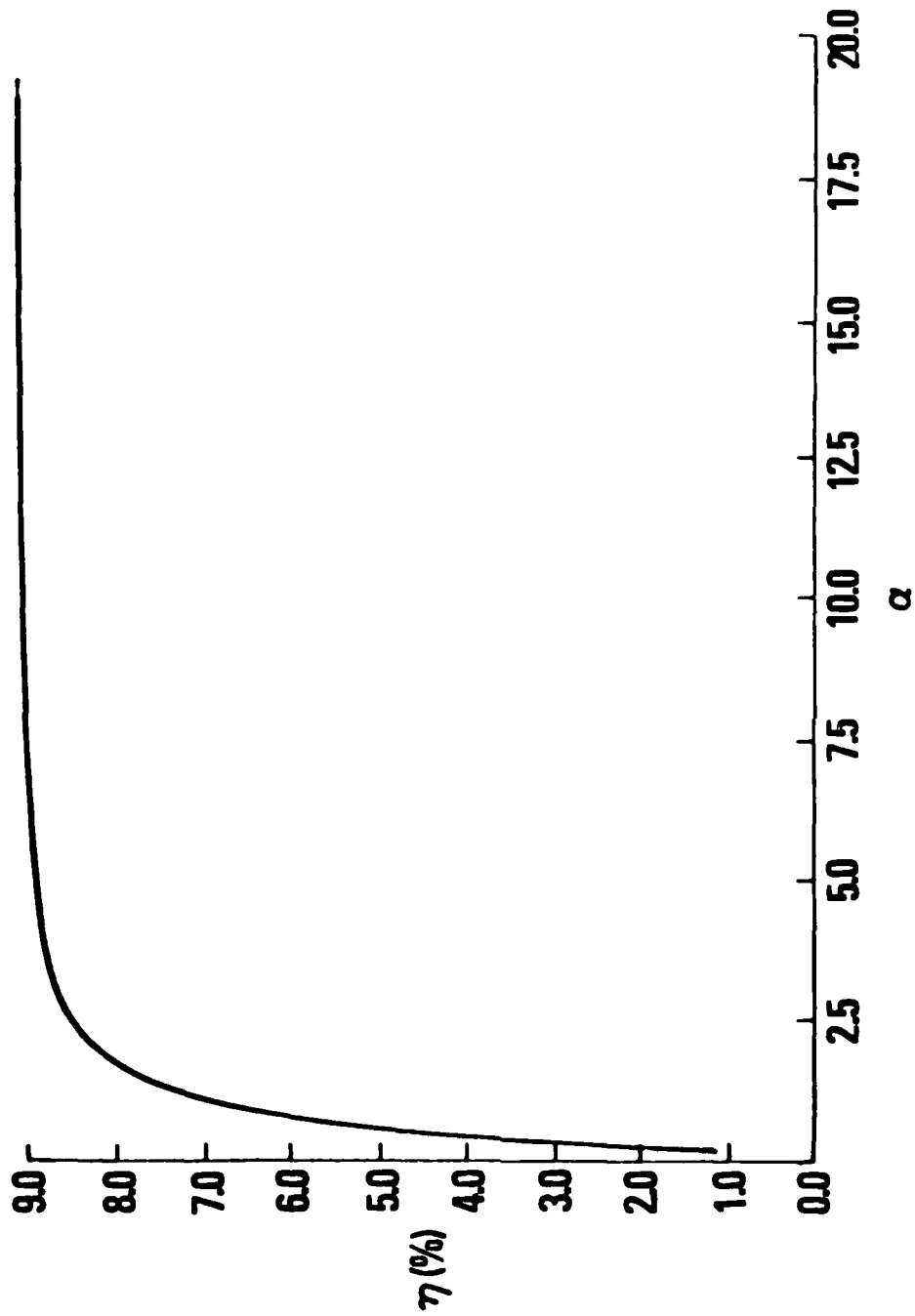


FIGURE 3-1. BEHAVIOR OF α , β_L , AND β_z AS FUNCTIONS OF R_L/R_W .

FIGURE 3-2. EFFICIENCY η AS A FUNCTION OF α .

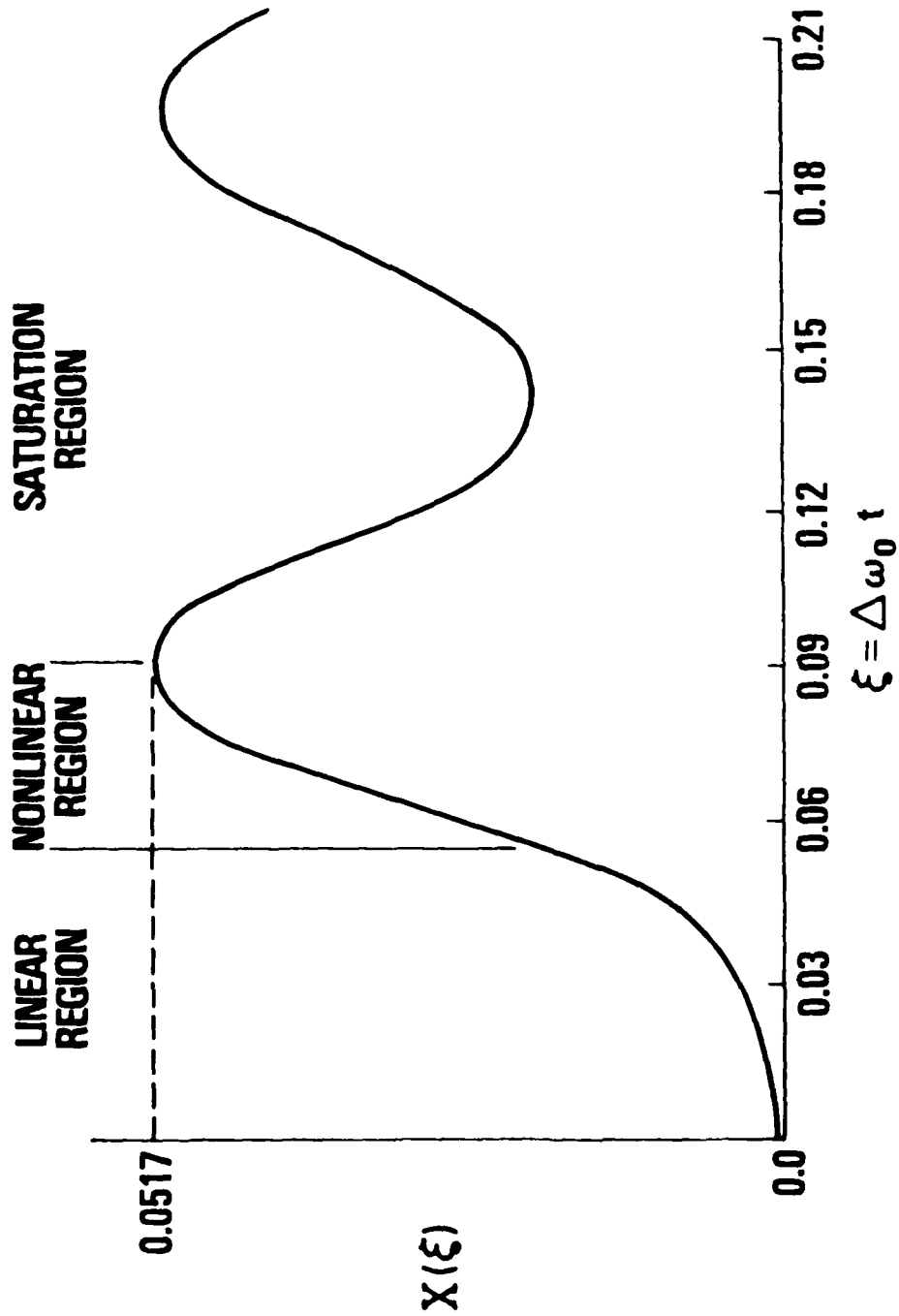


FIGURE 3-3. EVOLUTION OF THE WAVE FIELD AMPLITUDE. (FOR A 30.0 keV, 3.5 AMP BEAM IN A 460.0 GAUSS EXTERNAL MAGNETIC FIELD, THE SATURATION AMPLITUDE X_s IS 0.0517, CORRESPONDING TO A 9.11% EFFICIENCY.)

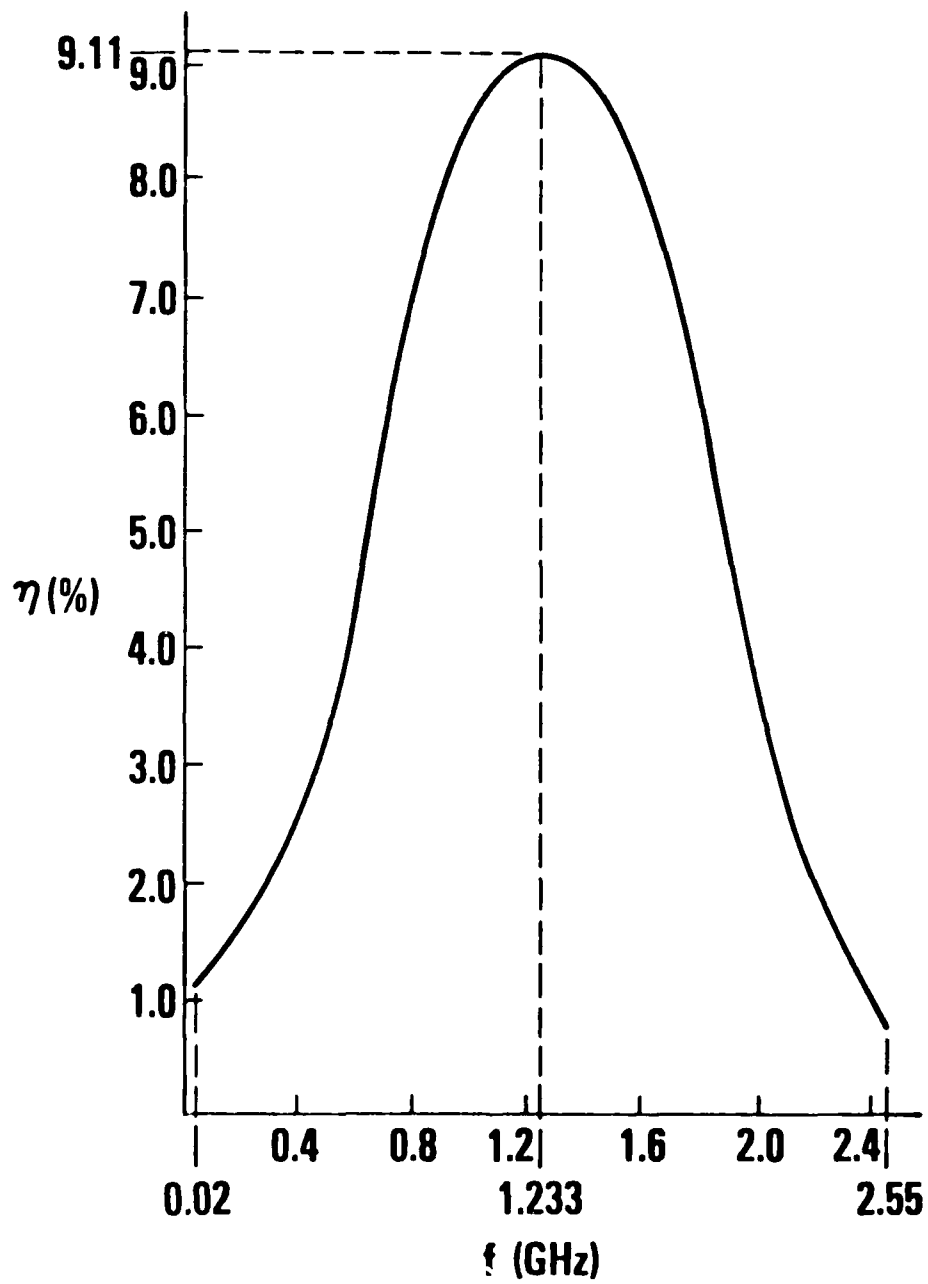


FIGURE 3-4. EFFICIENCY η AS A FUNCTION OF FREQUENCY \underline{f} ($\underline{f} = \omega/2\pi$). (THE EFFICIENCY IS PEAKED AT $\underline{f} = 1.233$ GHz.)

CHAPTER 4

SUMMARY AND CONCLUSIONS

SUMMARY OF RESULTS

We have derived a single nonlinear third-order differential equation (Equation (2-19)) to describe the evolution of the wave field amplitude in the TE_{11} interaction with a fast axis-rotating electron beam. From knowledge of the evolution of the wave field amplitude the efficiency of the interaction may be directly calculated. We assume phase trapping to be the dominant saturation mechanism. The derivation is self-consistent, with the effect of the electromagnetic fields on the electrons (phase averaged over the initial random distribution to obtain a collective effect) coupled to the response of the wave fields to the current induced by the electron bunching through conservation of energy.

Numerical solution of Equation (2-19) yields the wave field amplitude as a function of time over its entire range, through the linear, nonlinear and saturation regimes. The efficiency is directly calculated as a function of the experimental parameters beam energy, beam current, external magnetic field, and waveguide radius. The efficiency is also optimized with respect to wave frequency, with a resulting best efficiency of 9.11% at a frequency of $f \approx 1.233$ GHz.

APPROXIMATIONS AND ASSUMPTIONS

It is interesting to consider the implications of some of the approximations and assumptions made throughout the analysis. These are:

(a). The electromagnetic fields are taken to be vacuum waveguide fields with time dependent amplitudes.

The choice of vacuum waveguide electromagnetic fields with time-dependent amplitudes implies that the electron beam itself is tenuous; otherwise space charge effects would considerably distort the electromagnetic fields from their vacuum waveguide forms. This is a reasonable assumption to make for a 30.0 keV, 3.5 Amp beam, but would have to be modified in a high current situation.

(b). Only linear and nonlinear terms of the lowest order are retained in the performance of the phase averaging.

That the phase does not evolve reciprocally with the growth of the wave field is a pronounced drawback. We are currently exploring methods to include

higher order terms in our future work.

(c). Imposition of the grazing condition.

For the amplifier study, the grazing condition is expected to be valid at the point of greatest efficiency, but not elsewhere. The efficiency optimization study indicates that this is indeed the case. More tantalizing still are indications, both theoretical and experimental, that slightly off-grazing conditions may produce much higher efficiencies. The next extension of this work is to remove the grazing condition requirement. An additional advantage is that an oscillator study will then be possible, for which experimental data⁹ is available for direct comparison.

CONCLUSIONS

The accomplishments of FY87 are three. We have successfully developed the complete nonlinear analysis of the TE_{n1} interaction* with an axis-rotating electron beam, in a cylindrical waveguide, operated in the amplifier mode. We have performed, for the first time, a direct efficiency calculation for the TE_{11} mode interaction. We have, also for the first time, carried out an optimization with respect to wave frequency, an experimentally controllable parameter.

It should be remarked that the present method offers significant advantages in the numerical work involved as opposed to an undiluted particle simulation, by averaging out the fast time scale part of the Lorentz force. It is also anticipated that analytical studies of Equation (2-19), currently underway, will yield insight into the nature of the physical processes taking place.

* While the project was particularly directed toward the analysis of the TE_{n1} interaction in FY87, the formulation up to Equation (2-5) is general. The results shown in this report are easily extended to the TE_{n1} case. Studies of the TE_{21} and the TE_{31} interactions are in progress at the present time.

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APPENDIX A

EVALUATION OF $\langle f \sin \Phi \rangle$

We determine the phase averaged quantity $\langle f \sin \Phi \rangle$ in Equation (2-16) in terms of $E_o(t)$ and its derivatives using Equations (2-10) and (2-11). To include adequate information provided by the two coupled independent first-order differential equations a second-order differential equation is required. We therefore examine $d(f \sin \Phi)/dt$ and $d^2(f \sin \Phi)/dt^2$. Using Equations (2-10) and (2-11) to evaluate $\dot{\Phi}$, $\ddot{\Phi}$, $\dot{\alpha}$, and $\ddot{\alpha}$,

$$\frac{d(f \sin \Phi)}{dt} = -E_o c_1 \left(f' J_1' \sin^2 \Phi + f \frac{1}{\alpha} \frac{d(\alpha J_1')}{d\alpha} \cos^2 \Phi \right) - \Delta \omega f \cos \Phi, \quad (A-1)$$

$$\begin{aligned} \frac{d^2(f \sin \Phi)}{dt^2} = & -\Delta \omega^2 f \sin \Phi - E_o c_1 \left(f' J_1' \sin^2 \Phi + \frac{1}{\alpha} \frac{d}{d\alpha} (\alpha J_1') f \cos^2 \Phi \right) \\ & - \frac{d(\Delta \omega)}{dt} f \cos \Phi + E_o c_1 \left\{ 3 \Delta \omega \left(f' J_1' - \frac{1}{\alpha} \frac{d}{d\alpha} (\alpha J_1') \right) \right\} \sin \Phi \cos \Phi \\ & + E_o^2 c_1^2 \left\{ \left(f'' J_1'^2 + f' J_1' J_1'' \right) \sin^3 \Phi \right. \\ & + 3 \left(f' J_1' \left(\frac{1}{\alpha} \frac{d}{d\alpha} (\alpha J_1') \right) \right) \sin \Phi \cos^2 \Phi \\ & \left. + f \left(J_1' \left(\frac{d}{d\alpha} \left(\frac{1}{\alpha} \frac{d}{d\alpha} (\alpha J_1') \right) \right) - 2 \left(\frac{1}{\alpha} \frac{d}{d\alpha} (\alpha J_1') \right)^2 \right) \sin \Phi \cos^2 \Phi \right\}, \quad (A-2) \end{aligned}$$

where $f' = \alpha J_1''/\gamma + c_3 J_1'/\gamma^3$, $f'' = 2c_3 J_1''/\gamma^3 + (\alpha/\gamma) \{ J_1''' - 3c_3^2 (\Omega_o/\gamma_{11})^2 (J_1'/\gamma^4) \}$, and $c_3 = 1/\{1 - (v_{zo}/c)^2\}$. The dots \cdot and primes $'$ refer to differentiation with respect to time t and α respectively.

Two approximations are made. First, it is assumed that the effect of beam energy depletion can be neglected. This is consistent with the assumption that phase trapping is the dominant saturation mechanism. The consequences of assuming that $|\Delta\gamma(t)| \ll \gamma_o$ are: that $\gamma(t) \approx \gamma_o$ and $\alpha(t) \approx \alpha_o$. Second, it is assumed that the wave frequency is on the order of (but not equal to) the initial electron cyclotron frequency: $\omega \sim \omega_o$. With these two approximations, Equations (2-5), (A-1), and (A-2) become,

$$\Delta\omega \approx \Delta\omega_o - \left\{ c_4 \frac{f_o}{\gamma_o} \int_0^t dt' E_o(t') \sin\Phi(t') \right\}, \quad (A-3)$$

$$\begin{aligned} \frac{d(f \sin\Phi)}{dt} \approx & -\Delta\omega_o f_o \cos\Phi + c_4 \frac{(\alpha_o J'_{1o})^2}{\gamma_o^3} \int_0^t dt' E_o(t') \cos\Phi \sin\Phi' \\ & - c_1 E_o \left\{ \left(\frac{\alpha_o J'_{1o} J''_{1o}}{\gamma_o} + \frac{c_3 J'^2_{1o}}{\gamma_o^3} \right) \sin^2\Phi + \frac{J'_{1o}}{\gamma_o} \frac{d(\alpha_o J'_{1o})}{d\alpha_o} \cos^2\Phi \right\}, \end{aligned} \quad (A-4)$$

$$\begin{aligned} \frac{d^2(f \sin\Phi)}{dt^2} \approx & -\Delta\omega_o^2 f_o \sin\Phi + 2\Delta\omega_o c_4 \frac{(\alpha_o J'_{1o})}{\gamma_o^2} \int_0^t dt' E_o(t') f_o \sin\Phi \sin\Phi' \\ & - c_4^2 \frac{(\alpha_o J'_{1o})^2}{\gamma_o^2} \int_0^t dt' E_o(t') f_o \sin\Phi \int_0^{t'} dt'' E_o(t'') \sin\Phi' \sin\Phi'' \\ & - c_1 E_o \left\{ \left(\frac{\alpha_o J'_{1o} J''_{1o}}{\gamma_o} + \frac{c_3 J'^2_{1o}}{\gamma_o^3} \right) \sin^2\Phi + \left(\frac{J'^2_{1o}}{\gamma_o} + \frac{\alpha_o J'_{1o} J''_{1o}}{\gamma_o} \right) \cos^2\Phi \right\} \\ & + E_o \left\{ c_4 \frac{(\alpha_o J'_{1o})^2}{\gamma_o^3} - \frac{3c_1 J'^2_{1o}}{\gamma_o} \left(1 - \frac{c_3}{\gamma_o^2} \right) \Delta\omega_o \right\} \sin\Phi \cos\Phi \\ & c_1 E_o \left\{ \frac{3J'^2_{1o}}{\gamma_o^2} \left(1 - \frac{c_3}{\gamma_o^2} \right) c_4 \int_0^t dt' E_o(t') f_o \sin\Phi \cos\Phi \sin\Phi' \right\} \\ & + c_1^2 E_o^2 \left\{ \left(J'_{1o} J'''_{1o} + \frac{3c_3 J'_{1o} J''_{1o}}{\gamma_o^2} + J''_{1o} - 3c_3^2 \left(\frac{\Omega_o}{\gamma_{11} c} \right)^2 \frac{J'^2_{1o}}{\gamma_o^4} \right) f_o \sin^3\Phi \right. \\ & + c_1^2 E_o^2 \left\{ J''_{1o} + \frac{c_3 J'_{1o} J''_{1o}}{\alpha_o \gamma_o^2} + \frac{J'_{1o} J''_{1o}}{\alpha_o} + \frac{c_3 J'^2_{1o}}{\alpha_o \gamma_o^2} \right\} f_o \sin\Phi \cos^2\Phi \\ & \left. + c_1^2 E_o^2 \left\{ J'_{1o} \left(J'''_{1o} + \frac{J'_{1o}}{\alpha_o} - \frac{J'_{1o}}{\alpha_o^2} \right) - 2 \left(J''_{1o} + \frac{2J'_{1o} J''_{1o}}{\alpha_o} + \frac{J'_{1o}}{\alpha_o^2} \right) \right\} f_o \sin\Phi \cos^2\Phi \right\} \quad (A-5) \end{aligned}$$

where $c_4 = \left(\frac{\Omega_o}{\gamma_{11} c} \right)^2 c_2 (\omega^2 - k_z^2 c^2)$, and $f_o = f(\alpha_o)$, $J_{1o} = J_1(\alpha_o)$, etc.

The phase averages over the initial random phase distribution of the electron's transverse velocity are taken to obtain the collective effect of the wave field on the electrons. In Appendix B, explicit expressions for the phase averages of the combinations of trigonometric functions in Equations (A-3), (A-4),

and (A-5) are derived. A third approximation is made: only the linear and nonlinear terms of the lowest order are retained in the performance of the phase averaging. Substituting these approximate phase averages into Equations (A-3), (A-4), (A-5) and carrying out the necessary integrations one finds that, together with Equation (2-16),

$$\langle \Delta \omega \rangle = \Delta \omega_o - \frac{1}{2P\gamma_o} \{ E_o^2(t) - E_o^2(0) \}, \quad (A-6)$$

$$\frac{d \langle f \sin \Phi \rangle}{dt} = -\Delta \omega_o \langle f \sin \Phi \rangle - D_o I_s - D_1 E_o, \quad (A-7)$$

$$\frac{d^2 \langle f \sin \Phi \rangle}{dt^2} = -\{ \Delta \omega_o^2 + R_o E_o I_s - R_1 E_o^2 \} \langle f \sin \Phi \rangle \quad (A-8)$$

$$+ R_o E_o I_c \langle f \cos \Phi \rangle + D_o \{ \Delta \omega_o + \langle \Delta \omega \rangle \} I_c - D_1 E_o,$$

where

$$D_o = \frac{c_4}{2} \frac{(\alpha_o J'_{1o})^2}{\gamma_o^3},$$

$$D_1 = \frac{c_1}{2} \left\{ \frac{J_{1o}^2}{\gamma_o} \left(\frac{c_3}{\gamma_o^2} + 1 \right) + \frac{2\alpha_o J'_{1o} J''_{1o}}{\gamma_o} \right\},$$

$$R_o = \frac{3}{4} c_1 c_3 c_4 \left(\frac{\Omega_o}{\gamma_o \gamma_{11} c} \right)^2 \left(\frac{\alpha_o J'_{1o}}{\gamma_o} \right)^2,$$

$$R_1 = c_1^2 \left\{ \frac{1}{\alpha_o} \frac{d}{d\alpha_o} \left(J'_{1o} \frac{d(\alpha_o J'_{1o})}{d\alpha_o} \right) + \frac{3}{4} \left(\frac{\Omega_o}{\gamma_o \gamma_{11} c} \right)^2 c_3 \left[3 \left(\frac{\Omega_o}{\gamma_o \gamma_{11} c} \right)^2 (\alpha_o J'_{1o})^2 - 4 J'_{1o} \frac{d(\alpha_o J'_{1o})}{d\alpha_o} \right] \right\},$$

$$I_c = \int_0^t dt' E_o(t') \cos \langle \Delta \Phi(t - t') \rangle,$$

$$I_s = \int_0^t dt' E_o(t') \sin \langle \Delta \Phi(t - t') \rangle, \quad (A-9)$$

$$\text{with } \langle \Delta \Phi(t - t') \rangle = - \int_{t'}^t \langle \Delta \omega(t'') \rangle dt''.$$

APPENDIX B

EVALUATION OF PHASE AVERAGES

By using the half and double angle formulas we obtain the following exact and approximate expressions for the phase averages:

$$\begin{aligned}
 \langle \sin^2 \Phi \rangle &= \langle \frac{1}{2} (1 - \cos 2\Phi) \rangle = \frac{1}{2} , \\
 \langle \cos^2 \Phi \rangle &= \langle \frac{1}{2} (1 + \cos 2\Phi) \rangle = \frac{1}{2} , \\
 \langle \sin \Phi \cos \Phi \rangle &= \langle \frac{1}{2} \sin 2\Phi \rangle = 0 , \\
 \langle \sin^3 \Phi \rangle &= \langle \frac{1}{4} (3\sin \Phi + \sin 3\Phi) \rangle = \frac{3}{4} \langle \sin \Phi \rangle , \\
 \langle \sin \Phi \cos^2 \Phi \rangle &= \langle \sin \Phi (1 - \sin^2 \Phi) \rangle = \frac{1}{4} \langle \sin \Phi \rangle . \quad (B-1)
 \end{aligned}$$

Higher order terms involving $\langle \cos(2\Phi) \rangle$, $\langle \sin(2\Phi) \rangle$, and $\langle \sin(3\Phi) \rangle$ have been neglected.

Angle addition and subtraction laws are used to evaluate the following expressions

$$\begin{aligned}
 \langle \sin \Phi \sin \Phi' \rangle &= \langle \frac{1}{2} \{ \cos(\Phi - \Phi') - \cos(\Phi + \Phi') \} \rangle = \frac{1}{2} \cos \langle \Delta \Phi(t - t') \rangle , \\
 \langle \cos \Phi \sin \Phi' \rangle &= \langle \frac{1}{2} \{ -\sin(\Phi - \Phi') + \sin(\Phi + \Phi') \} \rangle = -\frac{1}{2} \sin \langle \Delta \Phi(t - t') \rangle , \\
 \langle \sin \Phi \cos \Phi \sin \Phi' \rangle &= \frac{1}{2} \{ \langle \sin \Phi (\cos \Phi \sin \Phi') \rangle + \langle \cos \Phi (\sin \Phi \sin \Phi') \rangle \} , \\
 &\approx -\frac{1}{4} \langle \sin \Phi \rangle \sin \langle \Delta \Phi(t - t') \rangle + \frac{1}{4} \langle \cos \Phi \rangle \cos \langle \Delta \Phi(t - t') \rangle . \quad (B-2)
 \end{aligned}$$

Higher order terms involving $\langle \cos(\Phi + \Phi') \rangle$ and $\langle \sin(\Phi + \Phi') \rangle$ have been neglected for the following reason. Based on Equation (2-11), Φ may be generally expanded⁵ as

$$\Phi = \langle \Phi \rangle + C_1 \sin(\langle \Phi \rangle + \theta_1) + C_2 \sin^2(\langle \Phi \rangle + \theta_2) + \dots \quad (\text{B-3})$$

where $\langle \Phi \rangle = \Phi_0 - \int_0^t \langle \Delta \omega(t') \rangle dt'$. Therefore

$$\Phi - \Phi' = -\int_t^t \langle \Delta \omega(t'') \rangle dt'' + \text{higher order terms} , \quad (\text{B-4})$$

$$\Phi + \Phi' = \text{higher order terms} ,$$

thus we approximate

$$\langle \Phi - \Phi' \rangle = \langle \Delta \Phi(t - t') \rangle \approx -\int_t^t \langle \Delta \omega(t'') \rangle dt'' . \quad (\text{B-5})$$

and neglect terms involving $\cos \langle \Phi + \Phi' \rangle$ and $\sin \langle \Phi + \Phi' \rangle$.

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